Empirical Energy Loss in Selected Binary Black Hole Mergers and Limitations of Mass-Ratio Scaling Models

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Abstract

We investigate the fraction of mass converted into gravitational wave energy during selected binary black hole mergers using published LIGO/Virgo parameter estimates. By analyzing representative events such as GW150914, GW151226, and GW190521, we obtain fractional mass-loss estimates of order 4%–6% of the total initial mass-energy.

We compare these estimates with a simplified mass-ratio-based scaling model, where the efficiency is assumed to depend only on the binary mass ratio. We find that this simplified scaling can underestimate the observed energy loss, illustrating the importance of strong-field general relativistic effects near merger, including nonlinear spacetime dynamics and ISCO physics.

Our results suggest an approximately common efficiency scale of order $\eta \approx 0.05$ among the selected events, while a larger analysis would be required to quantify dependence on mass ratio, spin, and other parameters. This provides a minimal empirical framework for a rough estimation of gravitational wave energy loss without full numerical relativity simulations.

1 Introduction

The direct detection of gravitational waves by the LIGO/Virgo collaboration has opened a new window into strong-field gravity and compact-object mergers [1, 2, 3]. Binary black hole (BBH) coalescences provide an important observational testbed for general relativity in the highly nonlinear regime [1, 4].

A useful derived quantity is the fraction of the initial binary mass-energy radiated as gravitational waves, estimated from published component and remnant-mass parameters [1, 2, 3]. Representative events such as GW150914, GW151226, and GW190521 indicate that a few percent of the initial binary mass can be emitted as gravitational radiation during merger [1, 5, 6].

While numerical relativity accurately models these systems, simplified analytical approaches remain useful for physical insight [4]. Here we consider a deliberately simplified mass-ratio-based scaling model and compare it with published event parameters.

Effects such as nonlinear spacetime dynamics near merger, the innermost stable circular orbit (ISCO), and relativistic plunge behavior are not captured by such a simple scaling law [7, 4]. As a result, discrepancies can arise between simple pedagogical estimates and values inferred from gravitational-wave parameter estimation.

Here, we compare selected BBH events with a simplified scaling estimate and find an approximately common radiated-energy fraction of order $\eta \approx 0.05$ for the events considered. This provides a compact empirical estimate for pedagogical use; more accurate modeling of BBH mergers still requires detailed waveform analysis or numerical relativity.

2 Methods

In this work, we estimate the mass-equivalent energy radiated in selected binary black hole (BBH) mergers using publicly available parameter estimates from gravitational wave observa-

tions reported by the LIGO/Virgo collaboration [2, 3]. We focus on events with published component-mass and remnant-mass estimates, allowing a simple estimate of the radiated energy using central parameter values.

2.1 Energy Loss Estimation

For each event, the total initial mass is defined as:

$$M_{\text{init}} = M_1 + M_2, \tag{1}$$

where M_1 and M_2 are the component black hole masses.

The final mass M_f is taken from gravitational wave parameter estimation results. The radiated energy is then computed using:

$$\Delta M = M_{\text{init}} - M_f, \tag{2}$$

and the corresponding energy loss fraction is given by:

$$\eta_{\text{obs}} = \frac{\Delta M}{M_{\text{init}}}. \tag{3}$$

2.2 Mass-Ratio Based Model

To compare the empirical estimates with a simplified mass-ratio-based scaling, we define a dimensionless mass ratio:

$$q = \frac{M_2}{M_1}, \qquad (M_2 \leq M_1). \tag{4}$$

We consider a phenomenological scaling model for the energy emission efficiency:

$$\eta_{\text{model}} = \eta_0 \cdot f(q), \tag{5}$$

where η_0 represents a fiducial efficiency scale and $f(q)$ is a dimensionless function of the mass ratio.

In this work, we test a simple scaling form:

$$f(q) = \frac{q^2}{(1+q)^4}, \tag{6}$$

which captures the symmetry suppression of highly unequal-mass binaries.

2.3 Comparison Framework

To compare the empirical estimates with the simplified model, we define the ratio:

$$R = \frac{\eta_{\text{model}}}{\eta_{\text{obs}}}. \tag{7}$$

This ratio provides a simple measure of how closely the chosen mass-ratio-based scaling reproduces the radiated-energy fractions inferred from published parameter estimates.

All calculations are performed using published parameter estimates from confirmed BBH merger events [1, 5, 6, 2, 3].

3 Results and Discussion

We consider selected confirmed binary black hole merger events reported by the LIGO/Virgo collaboration, including GW150914, GW151226, and GW190521 [1, 5, 6, 2, 3]. For each event, we estimate the total initial mass, final remnant mass, and the corresponding radiated energy fraction.

The results indicate that the fraction of mass converted into gravitational-wave energy falls in the approximate range of 4%–6% for the selected events, despite differences in total mass and mass ratio [1, 5, 6, 2, 3]. This suggests an approximately common efficiency scale for the representative events considered here.

When comparing these empirical estimates with the simplified mass-ratio scaling model, we find a noticeable mismatch. The scaling based solely on the mass-ratio function $f(q) = q^2/(1+q)^4$ can underestimate the observed radiated-energy fraction if used without additional calibration or strong-field merger physics. This illustrates the limitation of using mass ratio alone as the controlling parameter for radiated-energy efficiency.

This mismatch is consistent with the expectation that strong-field relativistic effects during the late inspiral and merger phases are essential for a complete description. In particular, nonlinear spacetime dynamics, relativistic plunge behavior, and full general relativistic interactions are not captured by simple mass-ratio scaling alone [7, 4].

4 Conclusion

In this work, we have estimated the mass-equivalent energy radiated in selected binary black hole mergers using published gravitational-wave parameter estimates. For the representative events considered here, the inferred radiated-energy fractions are of order $\eta \approx 0.05$, corresponding to a few percent of the initial binary mass [1, 5, 6, 2, 3].

The simplified mass-ratio-based scaling considered here is not sufficient by itself to reproduce these estimates, indicating that strong-field general relativistic effects are essential for describing the merger dynamics. A more complete description of gravitational wave energy emission therefore requires full nonlinear treatment of spacetime dynamics, as provided by numerical relativity.

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